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Relationship between Shehu transform with some other integral transform

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Abstract
Shehu transform is new integral transform type used to solve differential equations as other integral transforms. In this study, we discussed the relationship between this new integral transform with other some integral transforms.

Keyword: Shehu transform, ZZ transform, Mohand transform, Laplace transform, Sawi transform, Mahgoub transform.

Introduction

Many problems in engineering and science can be formulated in terms of differential equations. The ordinary differential equations arise in many areas of Mathematics, as well as in Sciences and Engineering. In order to solve the certain ordinary differential equations integral transforms are widely used. In this article we have construct the relation between Shehu transform and some other integral transforms which helps us to use Shehu transform simply in solving differential equations.

I. SHEHU TRANSFORM

Definition: A new transform called the Shehu transform of the function $f(t)$ belonging to a class A, where

$$A = \left\{ f(t) \colon \exists\, N, \eta_1, \eta_2 > 0, |f(t)| < N e^{\frac{|t|}{\eta_i}}, if\ t \in (-1)^i \times [0\ \infty) \right\}$$

Where $f(t)$ defined by $\mathbb{S}[f(t)]$ and is given by:

$$\mathbb{S}\{f(t)\} = W(s,u) = \int_0^\infty e^{\left(\frac{-st}{u}\right)} f(t)dt \qquad (1.1)$$

II. ZZ TRANSFORM

Let $f(t)$ be a function defined for all $t \geq 0$. The ZZ transform of $f(t)$ is the function $Z(u,s)$ defined by:

$$Z(u,s) = H\{f(t)\} = s\int_0^\infty f(ut)e^{-st}dt \qquad (1.2)$$

III. MOHAND TRANSFORM

Mohand transform of the function $f(t)$ $t \geq 0$ is given by:

$$M\{f(t)\} = H(s) = s^2\int_0^\infty f(t)e^{-st}dt \qquad (1.3)$$

IV. LAPLACE TRANSFORM

The Laplace transform of the function $f(t)$ $t \geq 0$ is given by:

$$L\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt \qquad (1.4)$$

V. MAHGOUB TRANSFORM

Mahgoub (Laplace-Carson) transform of the function $f(t)$ $t \geq 0$ is given by:

$$M_*\{f(t)\} = s\int_0^\infty f(t)e^{-st}dt \qquad (1.5)$$

$$0 < k_1 \leq s \leq k_2$$

VI. SAWI TRANSFORM

Sawi transform of the function $f(t)$ $t \geq 0$ is given by:

$$M_s\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(t)e^{\frac{-t}{s}}dt \qquad (1.6)$$

$$0 < k_1 \leq s \leq k_2$$

A. Connection between Shehu transform and ZZ transform

In this section we show that Shehu transform is theoretical dual of ZZ transform and the dual relation is given by the following relation:

Theorem 1.1: Let $f(t) \in A$ and if the Shehu transform and ZZ transform of $f(t)$ are $W(s,u)$and $Z(u,s)$ respectively then

$$Z(u,s) = \frac{s}{u}W(s,u) \qquad 1.7$$

And

$$\frac{u}{s}Z(u,s) = W(s,u) \qquad 1.8$$

Proof: From (1.2) we have

$$Z(u,s) = s\int_0^\infty f(ut)e^{-st}dt$$

Let $w = ut \Longrightarrow dt = \frac{dw}{u}$ from the above equation we have

$$Z(u,s) = s\int_0^\infty f(ut)e^{-st}dt$$

$$\Longrightarrow Z(u,s) = s\int_0^\infty f(w)e^{\frac{-sw}{u}}\frac{dw}{u}$$

$$\Longrightarrow Z(u,s) = \frac{s}{u}\int_0^\infty f(w)e^{\frac{-sw}{u}}dw$$

$$\Longrightarrow Z(u,s) = \frac{s}{u}W(s,u)$$

Hence the proof is completed

Now, to drive (1.8, from (1.1) we have:

$$\mathbb{S}\{f(t)\} = W(s,u) = \int_0^\infty e^{\left(\frac{-st}{u}\right)}f(t)dt$$

$$\Longrightarrow W(s,u) = \frac{u}{s}\left(\frac{s}{u}\int_0^\infty f(w)e^{\frac{-sw}{u}}dw\right)$$

Since, from (1.2) $\left(\frac{s}{u}\int_0^\infty f(w)e^{\frac{-sw}{u}}dw\right) = Z(u,s)$

$$\Longrightarrow W(s,u) = \frac{u}{s}Z(u,s)$$

Hence the proof of (1.8) is completed

Table 1: The relationship between Shehu transform and ZZ transform on some common functions.

$f(t)$	$\mathbb{S}\{f(t)\} = W(s,u)$	$H\{f(t)\} = Z(u,s)$	$\frac{u}{s}Z(u,s) = W(s,u)$
1	$\frac{u}{s}$	1	$\frac{u}{s}$
t	$\frac{u^2}{s^2}$	$\frac{u}{s}$	$\frac{u^2}{s^2}$
t^2	$\frac{2!\,u^3}{s^3}$	$\frac{2!\,u^2}{s^2}$	$\frac{2!\,u^3}{s^3}$
t^n	$\frac{n!\,u^{n+1}}{s^{n+1}}$	$\frac{n!\,u^n}{s^n}$	$\frac{n!\,u^{n+1}}{s^{n+1}}$
e^{at}	$\frac{u}{s-au}$	$\frac{s}{s-au}$	$\frac{u}{s-au}$
$\cos(at)$	$\frac{us}{s^2+\alpha^2u^2}$	$\frac{s^2}{s^2+\alpha^2u^2}$	$\frac{us}{s^2+\alpha^2u^2}$
$\sin(at)$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$	$\frac{aus}{s^2+(au)^2}$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$

B. Connection between Shehu transform and Laplace transform

Theorem 1.2: Let $f(t) \in A$ and $t \geq 0$ if the Shehu transform and Laplace transform of $f(t)$ are $W(s,u)$and $F(s)$ respectively then

$$W(s,u) = F\left(\frac{s}{u}\right) \qquad (1.9)$$

Proof: Since $W(s,u) = \frac{u}{s}Z(u,s) \Rightarrow W(s,u) = \frac{u}{s}\left(s\int_0^\infty f(ut)e^{-st}dt\right)$

$$\Rightarrow W(s,u) = \frac{u}{s}\left(s\int_0^\infty f(ut)e^{-st}dt\right)$$

$$\Rightarrow W(s,u) = u\int_0^\infty f(ut)e^{-st}dt$$

Put $w = ut \Rightarrow \frac{dw}{u} = dt$ in the above equation, we have

$$\Rightarrow W(s,u) = u\int_0^\infty f(t)e^{\frac{-sw}{u}}\frac{dw}{u}$$

$$\Rightarrow W(s,u) = \int_0^\infty f(t)e^{\frac{-sw}{u}}dw = F\left(\frac{s}{u}\right)$$

Therefore $W(s,u) = F\left(\frac{s}{u}\right)$

Hence the proof is completed

Table 2: The relationship between Shehu transform and Laplace transform of some common functions.

$f(t)$	$\mathbb{S}\{f(t)\} = W(s,u)$	$\mathcal{L}\{f(t)\} = F(s)$	$F\left(\frac{s}{u}\right) = W(s,u)$
1	$\frac{u}{s}$	$\frac{1}{s}$	$\frac{u}{s}$
t	$\frac{u^2}{s^2}$	$\frac{1}{s^2}$	$\frac{u^2}{s^2}$
t^2	$\frac{2!\,u^3}{s^3}$	$\frac{2!}{s^3}$	$\frac{2!\,u^3}{s^3}$
t^n	$\frac{n!\,u^{n+1}}{s^{n+1}}$	$\frac{n!}{s^{n+1}}$	$\frac{n!\,u^{n+1}}{s^{n+1}}$
e^{at}	$\frac{u}{s-au}$	$\frac{1}{s-a}$	$\frac{u}{s-au}$
$\cos(at)$	$\frac{us}{s^2+\alpha^2u^2}$	$\frac{s^3}{s^2+\alpha^2}$	$\frac{us}{s^2+\alpha^2u^2}$
$\sin(at)$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$	$\frac{\alpha s^2}{s^2+\alpha^2}$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$

C. Connection between Shehu transform and Mohand transform

Theorem 1.3: Let $f(t) \in A$ and $t \geq 0$, if the Shehu transform and Mohand transform of $f(t)$ are $W(s,u)$and $H(s)$ respectively then

$$\frac{u^2}{s^2}H\left(\frac{s}{u}\right) = W(s,u) \qquad 1.10$$

And
$$H\left(\frac{s}{u}\right) = \frac{s^2}{u^2}W(s,u) \qquad 1.11$$

Proof: Since, $W(s,u) = F\left(\frac{s}{u}\right)$

$$M\{f(t)\} = H(s) = s^2\int_0^\infty f(t)e^{-st}dt$$

$$\Longrightarrow M\{f(t)\} = s^2\left(\int_0^\infty f(t)e^{-st}dt\right)$$

$$\Longrightarrow M\{f(t)\} = H(s) = s^2F(s)$$

Now, if we substitute $s \longrightarrow \frac{s}{u}$

$$\Longrightarrow M\{f(t)\} = H\left(\frac{s}{u}\right) = \left(\frac{s}{u}\right)^2 F\left(\frac{s}{u}\right)$$

$$\Longrightarrow \frac{u^2}{s^2}M\{f(t)\} = \frac{u^2}{s^2}H\left(\frac{s}{u}\right) = F\left(\frac{s}{u}\right)$$

But, from (1.9) $W(s,u) = F\left(\frac{s}{u}\right)$ in the above equation we have

$$\Longrightarrow \frac{u^2}{s^2}H\left(\frac{s}{u}\right) = W(s,u)$$

Hence the proof is completed
Consequently, to drive (1.11)

$$\Longrightarrow \frac{u^2}{s^2}H(s) = W(s,u)$$

Now, multiply the above equation by $\frac{s^2}{u^2}$ both sides, we have

$$\Longrightarrow H\left(\frac{s}{u}\right) = \frac{s^2}{u^2}W(s,u)$$

Hence the proof is completed.

Table 3: The relationship between Shehu transform and Mohand transform of some common functions

$f(t)$	$\mathbb{S}\{f(t)\} = W(s,u)$	$M\{f(t)\} = H(s)$	$\frac{u^2}{s^2} H\left(\frac{s}{u}\right) = W(s,u)$
1	$\frac{u}{s}$	s	$\frac{u}{s}$
t	$\frac{u^2}{s^2}$	1	$\frac{u^2}{s^2}$
t^2	$\frac{2!\,u^3}{s^3}$	$\frac{2!}{s}$	$\frac{2!\,u^3}{s^3}$
t^n	$\frac{n!\,u^{n+1}}{s^{n+1}}$	$\frac{n!}{s^{n-1}}$	$\frac{n!\,u^{n+1}}{s^{n+1}}$
e^{at}	$\frac{u}{s-au}$	$\frac{s^2}{s-a}$	$\frac{u}{s-au}$
$\cos(at)$	$\frac{us}{s^2+\alpha^2u^2}$	$\frac{s^3}{s^2+\alpha^2}$	$\frac{us}{s^2+\alpha^2u^2}$
$\sin(at)$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$	$\frac{\alpha s^2}{s^2+\alpha^2}$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$

D. Connection between Shehu transform and MAHGOUB TRANSFORM

Theorem 1.4: Let $f(t) \in A$ and $t \geq 0$, if the Shehu transform and Mahgoub transform of $f(t)$ are $W(s,u)$and $G(s)$ respectively then

$$\frac{s}{u} W(s,u) = G\left(\frac{s}{u}\right) \qquad 1.12$$

And

$$W(s,u) = \frac{u}{s} G\left(\frac{s}{u}\right) \qquad 1.13$$

Proof: From (1.9) we have

$$M_*\{f(t)\} = s\int_0^\infty f(t)e^{-st}dt$$

$$\Longrightarrow M_*\{f(t)\} = s\left(\int_0^\infty f(t)e^{-st}dt\right)$$

$$\Longrightarrow M_*\{f(t)\} = G(s) = sF(s)$$

Now, if we substitute $s \longrightarrow \frac{s}{u}$

$$\Longrightarrow M_*\{f(t)\} = G\left(\frac{s}{u}\right) = \frac{s}{u} F\left(\frac{s}{u}\right)$$

Since from (1.9) $W(s,u) = F\left(\frac{s}{u}\right)$

$$\Longrightarrow G\left(\frac{s}{u}\right) = M_*\{f(t)\} = \frac{s}{u} W(s,u)$$

Therefore, $G\left(\frac{s}{u}\right) = \frac{s}{u} W(s,u)$. Hence, the proof of (1.12) is completed

Now, multiply the above equation by $\frac{u}{s}$ both sides, we have:

$$\Rightarrow W(s,u) = \frac{u}{s} G\left(\frac{s}{u}\right)$$

Hence the proof of (1.13) is completed

Table 4: The relationship between Shehu transform and Mohgoub transform of some common functions.

$f(t)$	$\mathbb{S}\{f(t)\} = W(s,u)$	$M_*\{f(t)\} = G(s)$	$\frac{u}{s} G\left(\frac{s}{u}\right) = W(s,u)$
1	$\frac{u}{s}$	1	$\frac{u}{s}$
t	$\frac{u^2}{s^2}$	$\frac{1}{s}$	$\frac{u^2}{s^2}$
t^2	$\frac{2!\,u^3}{s^3}$	$\frac{2!}{s^2}$	$\frac{2!\,u^3}{s^3}$
t^n	$\frac{n!\,u^{n+1}}{s^{n+1}}$	$\frac{n!}{s^n}$	$\frac{n!\,u^{n+1}}{s^{n+1}}$
e^{at}	$\frac{u}{s-au}$	$\frac{s}{s-a}$	$\frac{u}{s-au}$
$\cos(at)$	$\frac{us}{s^2+\alpha^2u^2}$	$\frac{s^2}{s^2+\alpha^2}$	$\frac{us}{s^2+\alpha^2u^2}$
$\sin(at)$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$	$\frac{\alpha s}{s^2+\alpha^2}$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$

E. Connection between Shehu transform and SAWI TRANSFORM

Theorem 1.5: Let $f(t) \in A$ and $t \geq 0$, if the Shehu transform and Sawi transform of $f(t)$ are $W(s,u)$and $J(s)$ respectively then

$$J\left(\frac{s}{u}\right) = \left(\frac{u}{s}\right)^2 W(s,u)$$

Proof: From equation (1.6) we have

$$M_s\{f(t)\} = J(s) = \frac{1}{s^2}\int_0^\infty f(t)e^{\frac{-t}{s}}dt$$

$$\Rightarrow M_s\{f(t)\} = J(s) = \left(\frac{1}{s}\right)^2\int_0^\infty f(t)e^{\frac{-t}{s}}dt$$

$$\Rightarrow M_s\{f(t)\} = J(s) = \left(\frac{1}{s}\right)^2\left(\int_0^\infty f(t)e^{\frac{-t}{s}}dt\right)$$

$$\Rightarrow M_s\{f(t)\} = J(s) = \left(\frac{1}{s}\right)^2 F\left(\frac{1}{s}\right)$$

Now, if we substitute $s \longrightarrow \frac{u}{s}$

$$\Rightarrow M_s\{f(t)\} = J\left(\frac{u}{s}\right) = \left(\frac{s}{u}\right)^2 F\left(\frac{s}{u}\right) = \left(\frac{s}{u}\right)^2 W(s,u)$$

$$\Rightarrow M_s\{f(t)\} = J\left(\frac{s}{u}\right) = \left(\frac{s}{u}\right)^2 W(s,u)$$

Hence, the proof is completed

Table 5: The relationship between Shehu transform and Sawi transform of some common functions.

$f(t)$	$\mathbb{S}\{f(t)\} = W(s,u)$	$M_s\{f(t)\} = J(s)$	$\frac{u^2}{s^2} J\left(\frac{s}{u}\right) = W(s,u)$
1	$\frac{u}{s}$	$\frac{1}{s}$	$\frac{u}{s}$
t	$\frac{u^2}{s^2}$	1	$\frac{u^2}{s^2}$
t^2	$\frac{2!\,u^3}{s^3}$	$2!\,s$	$\frac{2!\,u^3}{s^3}$
t^n	$\frac{n!\,u^{n+1}}{s^{n+1}}$	$n!\,s^{n-1}$	$\frac{n!\,u^{n+1}}{s^{n+1}}$
e^{at}	$\frac{u}{s-au}$	$\frac{1}{s(1-as)}$	$\frac{u}{s-au}$
$\cos(at)$	$\frac{us}{s^2+\alpha^2u^2}$	$\frac{1}{s(1+s^2\alpha^2)}$	$\frac{us}{s^2+\alpha^2u^2}$
$\sin(at)$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$	$\frac{\alpha}{1++\alpha^2s^2}$	$\frac{\alpha u^2}{s^2+\alpha^2u^2}$

Conclusion

In this paper, we have successfully discussed the relationship between Shehu transform and some other integral transforms. We have also used tabular representation of Shehu transform and some other integral transform on some common functions to show the connection between Shehu transform and some other integral transform namely, ZZ transform, Mohand transform, Laplace transform, Sawi transform, Mahgoub transform.

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